

## Notes 2.3 – Intro to Properties of Logarithms

Warmup – Find the value for each logarithm.

1.  $\log_5 625 = 4$

2.  $\log_5 0.2 = -1$

3.  $\log_x x^7 = 7$

Answer the questions about each graph.

4. What is the value of  $x$  when  $f(x) = 0$ ?

$$x = 1$$

What is the value of  $x$  when  $f(x) = 1$ ?

$$x = 2$$

What is the value of  $f(x)$  when  $x = 2$ ?

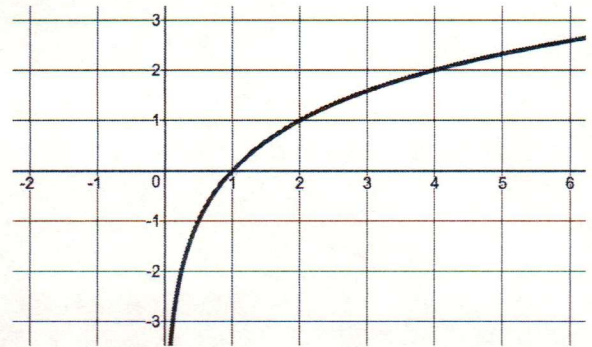
$$y = 1$$

What will be the value of  $x$  when  $f(x) = 4$ ?

$$x = 16$$

What is the equation of the graph?

$$f(x) = \log_2 x$$

5. What is the value of  $x$  when  $f(x) = 0$ ?

$$x = 1$$

What is the value of  $x$  when  $f(x) = 1$ ?

$$x = 3$$

What is the value of  $f(x)$  when  $x = 9$ ?

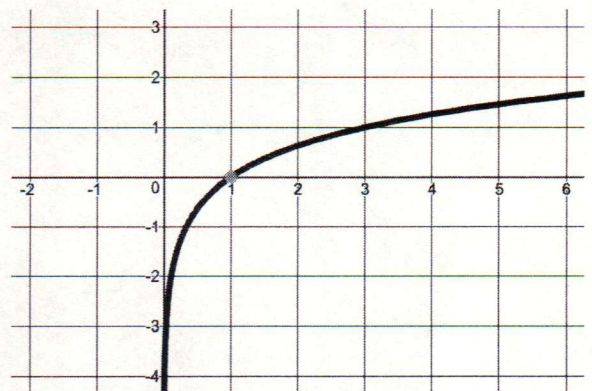
$$y = 2$$

What will be the value of  $x$  when  $f(x) = 4$ ?

$$x = 81$$

What is the equation of the graph?

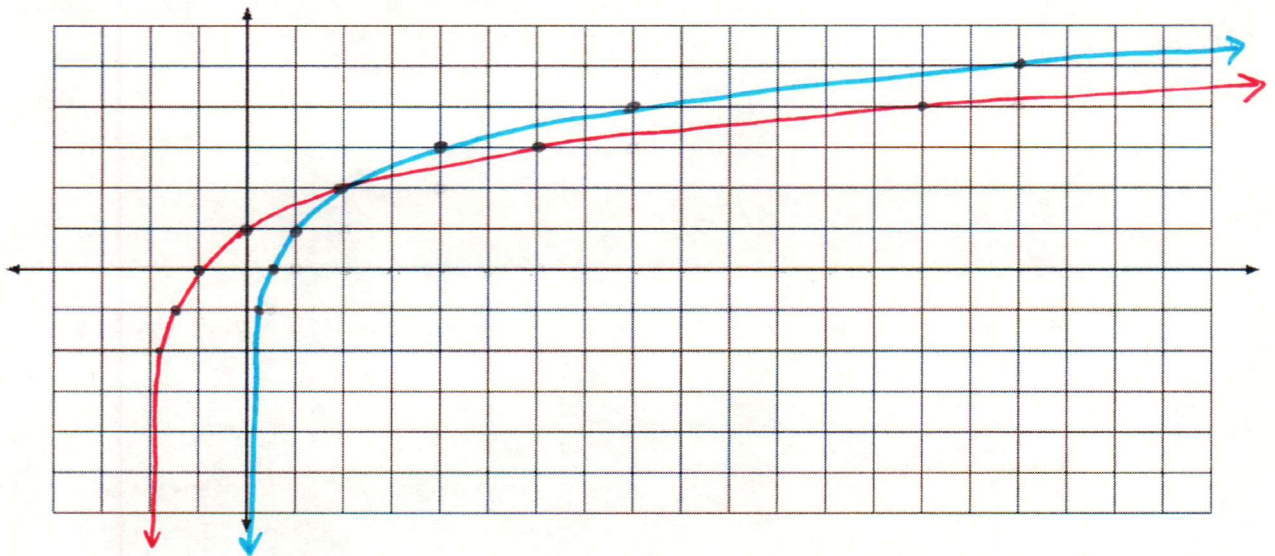
$$f(x) = \log_3 x$$



### Investigation

Graph the function  $f(x) = \log_2(x+2)$

$x$	$-1\frac{1}{2}$	$-1$	$0$	$2$	$6$	$14$
$y$	$-1$	$0$	$1$	$2$	$3$	$4$



Can we distribute the logarithm across the parenthesis to be  $f(x) = \log_2 x + \log_2 2$ ? How can we check if that works?

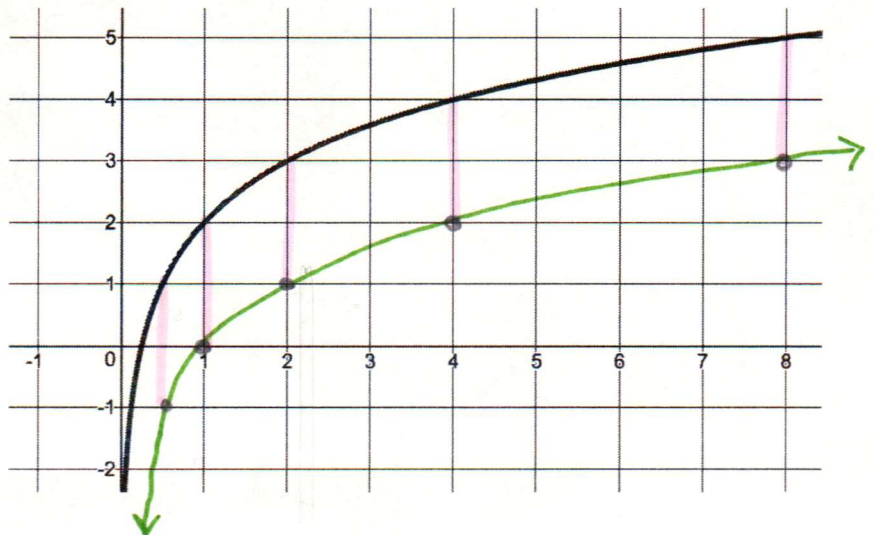
- make tables } then
- graph both } compare

$x$	$\frac{1}{2}$	$1$	$2$	$4$	$8$	$16$
$y$	$0$	$1$	$2$	$3$	$4$	$5$

They are not the same thing.

The graph below is the function  $f(x) = \log_2(4x)$ . Draw the graph of  $f(x) = \log_2 x$  on the same graph. What do you notice?

The graph is a vertical shift up of 2.

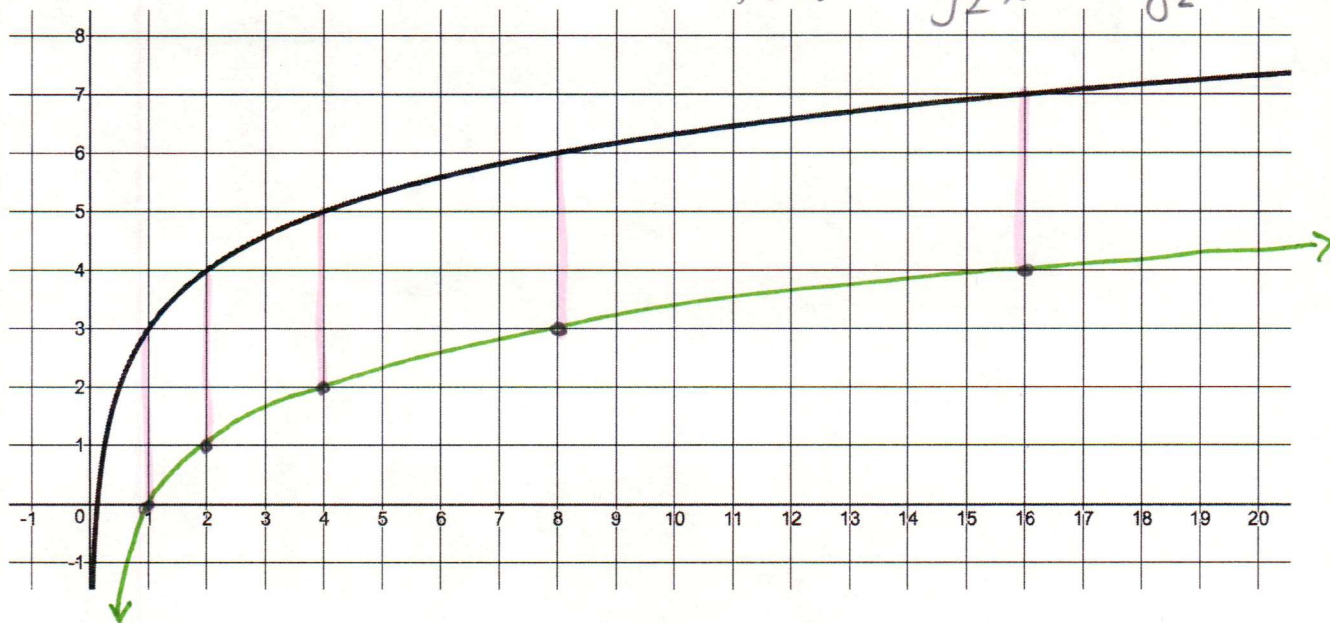


or  $f(x) = \log_2 x + 2$  gives the same graph,  
 $f(x) = \log_2 x + \log_2 4$



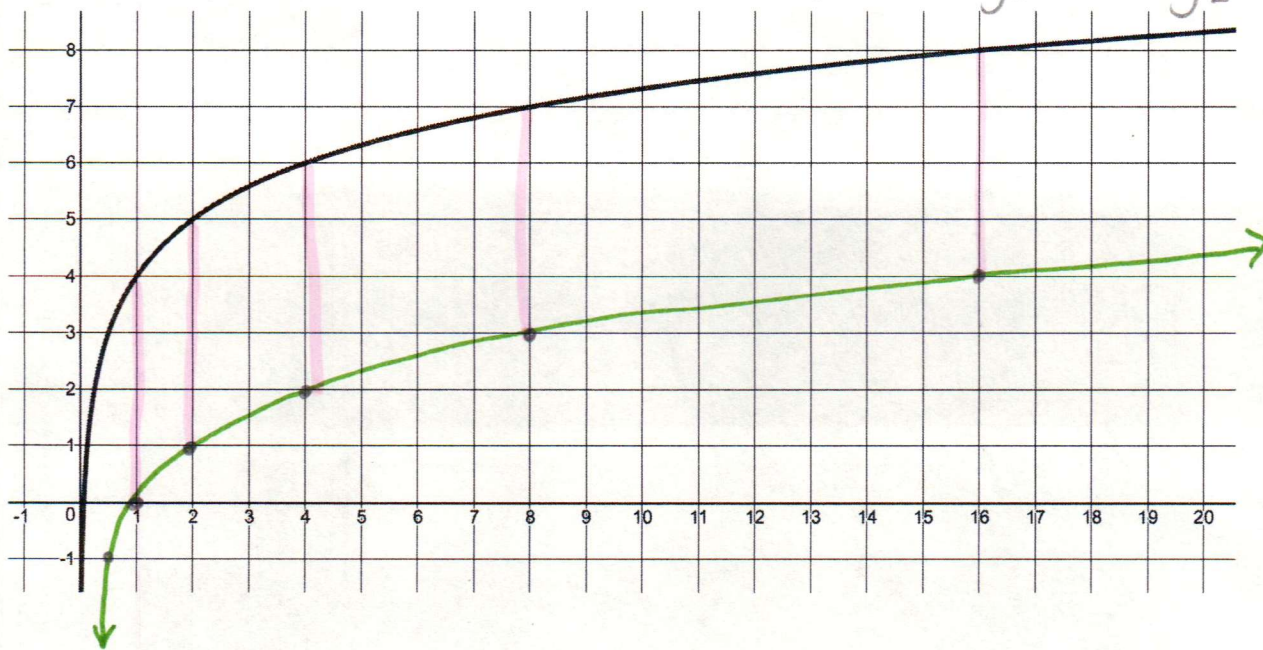
$f(x) = \log_2(8x)$  is below.

Equivalent equation:  $f(x) = \log_2 x + 3$   
 $f(x) = \log_2 x + \log_2 8$



$f(x) = \log_2(16x)$  is below.

Equivalent equation:  $f(x) = \log_2 x + 4$   
 $f(x) = \log_2 x + \log_2 16$



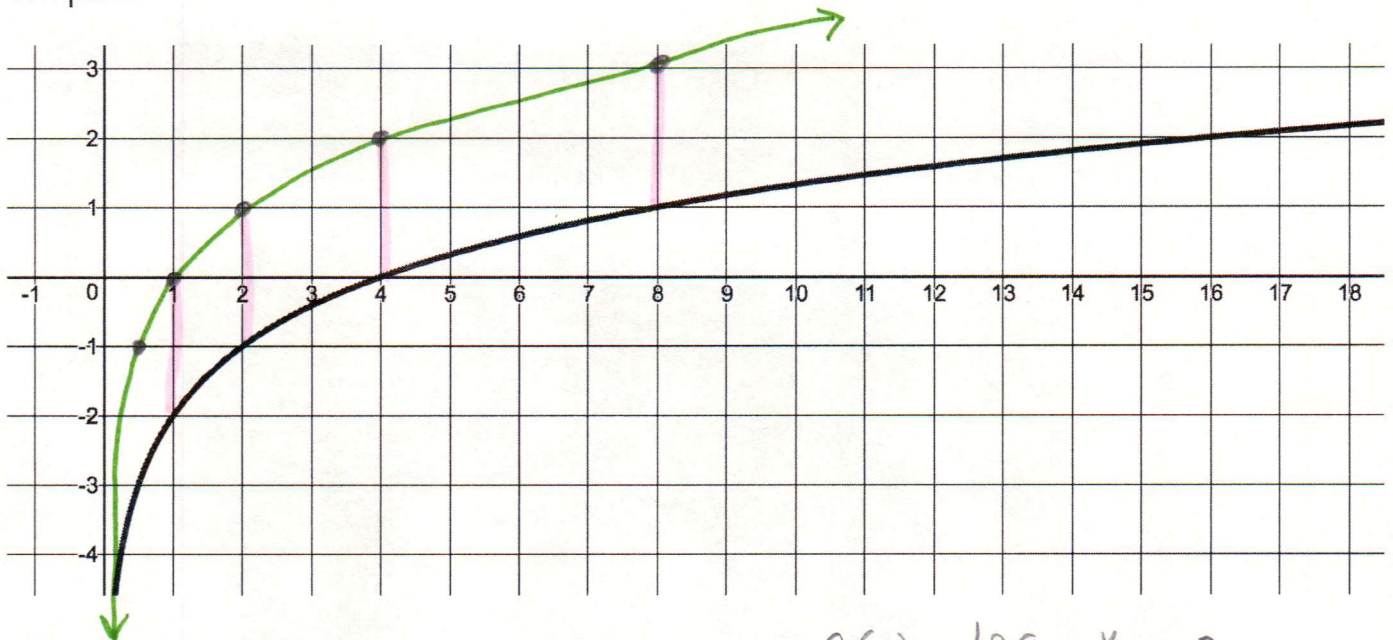
How can we write a log in the form  $f(x) = \log_b(mn)$  as the sum of two logs?

$f(x) = \log_b(mn)$  becomes  $f(x) = \log_b m + \log_b n$



Do you think that dividing inside a log would have a similar relationship?

Below is the graph of  $f(x) = \log_2\left(\frac{x}{4}\right)$ , draw the graph of  $f(x) = \log_2 x$  on the same graph and compare.



What do you notice?

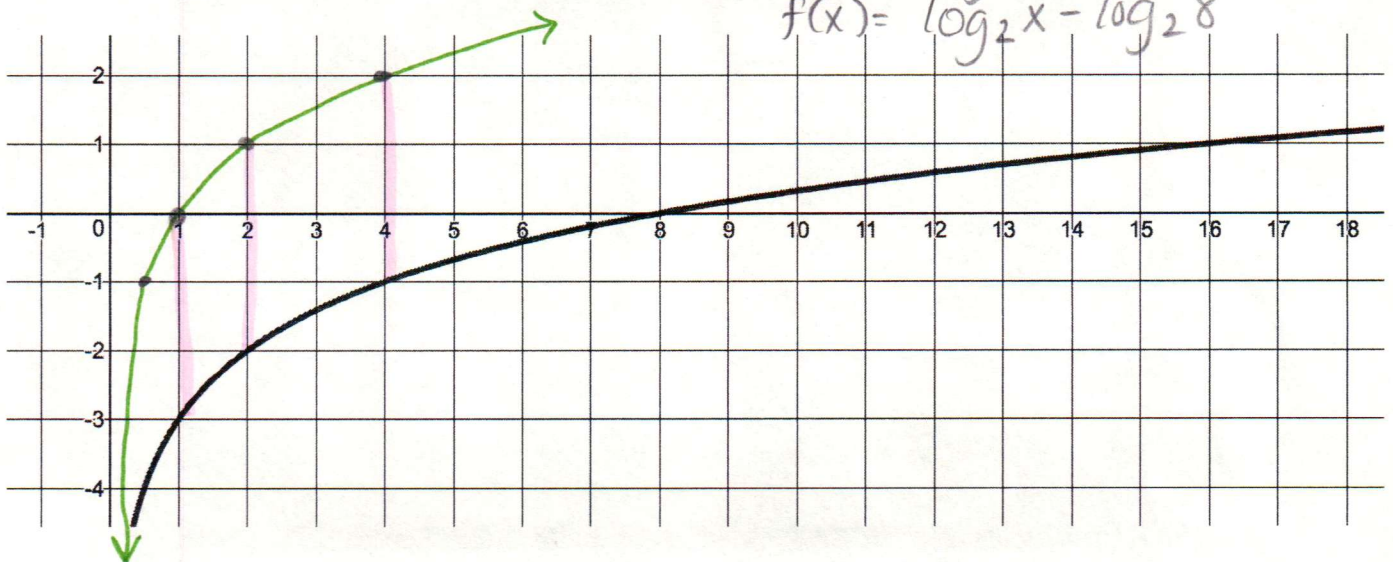
The graph shifts down instead of up

$$f(x) = \log_2 x - 2$$

$$f(x) = \log_2 x - \log_2 4$$

$f(x) = \log_2\left(\frac{x}{8}\right)$  is below.

Equivalent equation:  $f(x) = \log_2 x - 3$   
 $f(x) = \log_2 x - \log_2 8$



What is the general form for division inside a logarithm?

$f(x) = \log_b\left(\frac{m}{n}\right)$  becomes

$$f(x) = \log_b m - \log_b n$$

Using what we learned about multiplying within a logarithm, how can you rewrite this logarithm?

$$f(x) = \log_2(x^3) \rightarrow f(x) = \log_2(x \cdot x \cdot x)$$

$$\rightarrow f(x) = \log_2 x + \log_2 x + \log_2 x$$

$$\rightarrow \boxed{f(x) = 3 \cdot \log_2 x}$$

### Logarithm Properties

Property	Rule
Log of a Product	$\log_b(\underline{mn}) = \log_b m + \log_b n$
Log of a Quotient	$\log_b(\underline{\frac{m}{n}}) = \log_b m - \log_b n$
Log of a Power	$\log_b(\underline{m^n}) = n \cdot \log_b m$

How are log properties similar to the rules for exponents?

when multiplying with a common base, add the exponents  
when dividing with a common base, subtract the exponents  
when raising to a power, multiply the exponents